CBSE Class 9 Mathemaics

Important Questions

Chapter 10

Circles

1 Marks Quetions

1. An angle in the semicircle is

(a) Right angle

(b) 180°

(c) 360°

(d) none of these

Ans. (a) Right angle

2. If the angles subtended by two chords of a circle at the centre are equal then the chords are

(a) not equal

(b) equal

(c) angle equal

(d) line equals

Ans. (b) equal

3. How many circle passing through three non-collinear points

(a) one

(b) two

(c) three

(d) four

Ans. (a) one

4. The constant distance is called

(a) diameter

(b) radius

(c) centre

(d) circle

Ans. b) radius

5. PS and RS are two chord's of a circle such that PQ=10cm and RS= 24cm and PQ||RS. The distance between PQ and RS is 17cm. Find the radius of circle

(a) 10cm

(b) 13cm

(c) 15cm

(d) none of these

Ans. (b) 13cm

6. A circle is drawn. It divides the plane into

(a) 3 Parts

(b) 4 Parts

(c) 5 Parts

(d) No Parts

Ans. (a) 3 Parts

7. The relation between diameter and radius of a circle is

(a) r=2d

(b) d=r

(c) d=2r

(d) d=2 π r

Ans. (c) d=2r

8. If P and Q are any two Points on a circle then PQ is called a

(a) diameter

(b) secant

(c) chord

(d) radius

Ans. c) chord

9. What is a diameter

(a) $r = 2d$

(b) $d = 2\pi r$

(c) $d = r$

(d) $d = 2r$

Ans. (d) $d = 2r$

10. Two point on a circle shows the

(a) radius

(b) chord

(c) secant

(d) diameters

Ans. b) chord

11. The whole arc of a circle is called

(a) circumference

(b) semi-circle

(c) sector

(d) segment

Ans. (a) circumference

12. One half of the whole arc of a circle

(a) semi-circle

(b) circumference

(c) segment

(d) sector

Ans. (a) semi-circle

13. Circle having same centre are said to be

(a) Concentric

(b) circle

(c) chord

(d) secant

Ans. (a) Concentric

14. The line which meet a circle in two points is called a

(a) chord of circle

(b) diameter

(c) radius

(d) secant of circle

Ans. (d) secant of circle

15. The sum of either pair of opposite angle of cyclic quadrilateral is

(a) $360°$

(b) $90°$

(c) $180°$

(d) $270°$

Ans. c) $180°$

16. Two circle are congruent if they have equal.

(a) diameter

(b) radius

(c) chord

(d) secant

Ans. (b) radius

17. Which equation is show the diameter of circle

(a) $d = 2r$
(b) $d = r$
(c) $d = 2\pi r$
(d) $r = 2d$

Ans. (a) $d = 2r$

18. $\frac{1}{2}$ **of the whole circle shows**

(a) semi-circle

(b) circumference

(c) sector

(d) segments

Ans. (a) semi-circle

19. Two circle are congruent if they have equal

(a) radius

(b) diameter

(c) chord

(d) secant

Ans. (a) radius

CBSE Class 9 Mathemaics

Important Questions

Chapter 10

Circles

2 Marks Quetions

1. Fill in the blanks:

(i) The centre of a circle lies in ______________ of the circle.

(ii) A point, whose distance from the centre of a circle is greater than its radius lies in ______________ of the circle.

(iii) The longest chord of a circle is a ______________ of the circle.

(iv) An arc is a ______________ when its ends are the ends of a diameter.

(v) Segment of a circle is the region between an arc and ______________ of the circle.

Ans. (i) Interior

(ii) Exterior

(iii) diameter

(iv) Semi-circle

(v) Chord

(vi) Three

2. Write True or False:

(i) Line segment joining the centre to any point on the circle is a radius of the circle.

(ii) A circle has only finite number of equal chords.

(iii) If a circle is divided into three equal arcs each is a major arc.

(iv) A chord, which is twice as long as its radius is a diameter of the circle.

(v) Sector is the region between the chord and its corresponding arc.

(vi) A circle is a plane figure.

Ans. (i) True

(ii) False

(iii) False

(iv) True

(v) False

(vi) True

3. If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chord.

Ans. Given: AB and CD be two equal chords of a circle with centre O intersecting each other with in the circle at point E. OE is joined.

To prove: $\angle$ OEM = $\angle$ OEN

Construction: Draw OM $\perp$ AB and ON $\perp$ CD.

Proof: In right angled triangles OME and ONE,

$\angle$ OME = $\angle$ ONE [Each 90°]

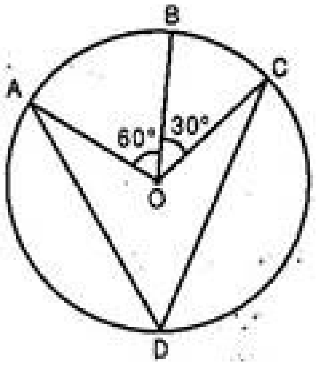
OM = ON [Equal chords are equidistant from the centre]

OE = OE [Common]

$\therefore$ $\triangle$OME $\cong$ $\triangle$ONE [RHS rule of congruency]

$\therefore$ $\angle$ OEM = $\angle$ OEN [By CPCT]

4. In figure, A, B, C are three points on a circle with centre O such that $\angle$ BOC = 30°. $\angle$ AOB = 60°. If D is a point on the circle other than the arc ABC, find $\angle$ ADC.



Ans. $\angle$ AOC = $\angle$ AOB + $\angle$ BOC $\Rightarrow$ $\angle$ AOC = $60^\circ + 30^\circ = 90^\circ$

Now $\angle$ AOC = 2 $\angle$ ADC

[$\because$ Angled subtended by an arc, at the centre of the circle is double the angle subtended by the same arc at any point in the remaining part of the circle]

$\Rightarrow$ $\angle$ ADC = $\frac{1}{2}$ $\angle$ AOC

$\Rightarrow$ $\angle$ ADC = $\frac{1}{2} \times 90^\circ$ = 45°

5. In figure, $\angle$ PQR = 100°, where P, Q, R are points on a circle with centre O. Find $\angle$ OPR.

Ans. In the figure, Q is a point in the minor arc $\widehat{PQR}$.

$\therefore$ $m\widehat{RP}$ = 2 $\angle$ PQR $\Rightarrow$ $\angle$ ROP = 2 $\angle$ PQR

$\Rightarrow$ $\angle$ ROP = $2 \times 100^\circ$ = 200°

Now $m\widehat{PR} + m\widehat{RP} = 360^\circ$ $\Rightarrow$ $\angle$ POR + $\angle$ ROP = 360°

$\Rightarrow$ $\angle$ POR + 200° = 360° $\Rightarrow$ $\angle$ POR = $360^\circ - 200^\circ$ = 160° .....(i)

Now ΔOPR is an isosceles triangle.

$\therefore$ OP = OR [radii of the circle]

$\Rightarrow$ $\angle$OPR = $\angle$ORP [angles opposite to equal sides are equal] .....(ii)

Now in isosceles triangle OPR,

$\angle$OPR + $\angle$ORP + $\angle$POR = 180°

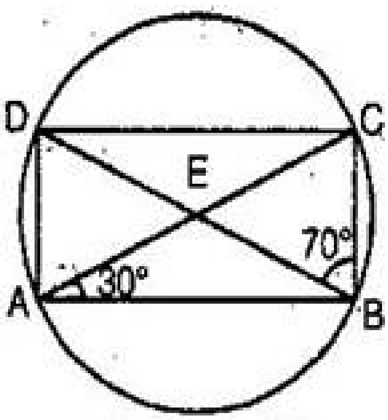
$\Rightarrow$ $\angle$OPR + $\angle$ORP + 160° = 180°

$\Rightarrow$ 2$\angle$OPR = $180^\circ - 160^\circ$ [Using (i) & (ii)]

$\Rightarrow$ 2$\angle$OPR = 20°

$\Rightarrow$ $\angle$OPR = 10°

6. ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. $\angle$DBC = 70°, $\angle$BAC is 30° find $\angle$BCD. Further if AB = BC, find $\angle$ECD.



Ans. Here, $\angle$DBC = 70° and $\angle$BAC = 30°

And $\angle$DAC = $\angle$DBC = 70° [Angles in same circle]

Now ABCD is a cyclic quadrilateral.

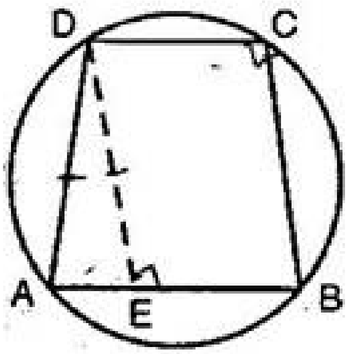
$\therefore$ $\angle$DAB + $\angle$BCD = 180°

[Sum of opposite angles of a cyclic quadrilateral is supplementary]

$\Rightarrow$ $100^\circ + \angle$BCD = 180°

$\Rightarrow \angle BCD = 80^\circ$

7. If the non-parallel sides of a trapezium are equal, prove that it is cyclic.



Ans. Given: A trapezium ABCD in which AB $\parallel$ CD and AD = BC.

To prove: The points A, B, C, D are concyclic.

Construction: Draw DE $\parallel$ CB.

Proof: Since DE $\parallel$ CB and EB $\parallel$ DC.

$\therefore$ EBCD is a parallelogram.

$\therefore$ DE = CB and $\angle DEB = \angle DCB$

Now AD = BC and DA = DE

$\Rightarrow \angle DAE = \angle DEB$

But $\angle DEA + \angle DEB = 180^\circ$

$\Rightarrow \angle DAE + \angle DCB = 180^\circ$ [$\because \angle DEA = \angle DAE$ and $\angle DEB = \angle DCB$]

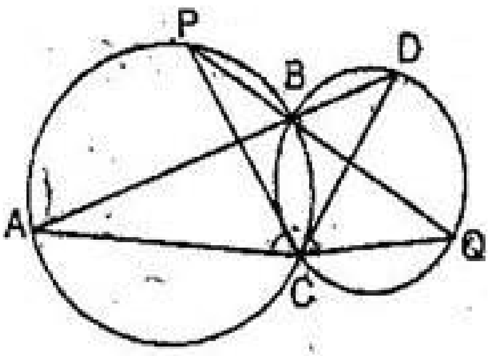
$\Rightarrow \angle DAB + \angle DCB = 180^\circ$

$\Rightarrow \angle A + \angle C = 180^\circ$

Hence, ABCD is a cyclic trapezium.

8. Two circles intersect at two points B and C. Through B, two line segments ABD and

PBQ are drawn to intersect the circles at A, D, P, Q respectively (see figure). Prove that ∠ACP = ∠QCD.



Ans. In triangles ACD and QCP,

∠A = ∠P and ∠Q = ∠D [Angles in same segment]

∴ ∠ACD = ∠QCP [Third angles] ....(i)

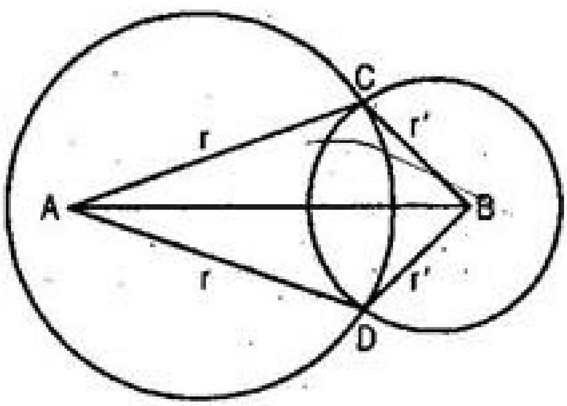
Subtracting ∠PCD from both the sides of eq. (i), we get,

∠ACD - ∠PCD = ∠QCP - ∠PCD

⇒ ∠ACPO = ∠QCD

Hence proved.

9. Prove that the line of centres of two intersecting circles subtends equal angles at the two points of intersection.



Ans. Let two circles with respective centers A and B intersect each other at points C and D.

We have to prove $\angle ACB = \angle ADB$

Proof: In triangles ABC and ABD,

AC = AD = r

BC = BD = r

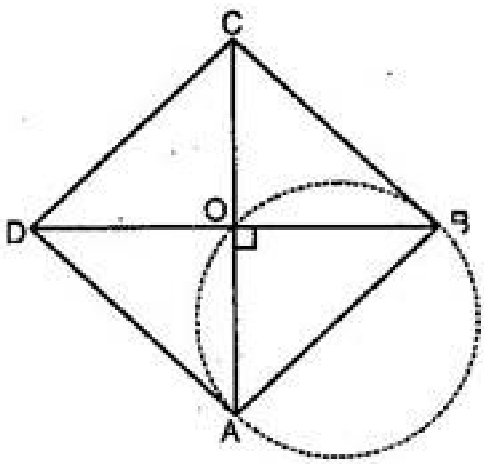
AB = AB [Common]

$\therefore \triangle ABC \cong \triangle ABD$

[SSS rule of congruency]

$\Rightarrow \angle ACB = \angle ADB$ [By CPCT]

10. Prove that the circle drawn with any drawn with any side of a rhombus as a diameter, passes through the point of intersection of its diagonals.



Ans. Let ABCD be a rhombus in which diagonals AC and BD intersect each other at point O.

As we know that diagonals of a rhombus bisect and perpendicular to each other.

$\therefore \angle AOB = 90^\circ$

And if we draw a circle with side AB as diameter, it will definitely pass through point O (the point intersection of diagonals) because then $\angle AOB = 90^\circ$ will be the angle in a semi-circle.

11. AB = DC and diagonal AC and BD intersect at P in cyclic quadrilateral Prove that

$\Delta PAB \cong \Delta PDC$

Ans. In Δ PAB and ΔPDC

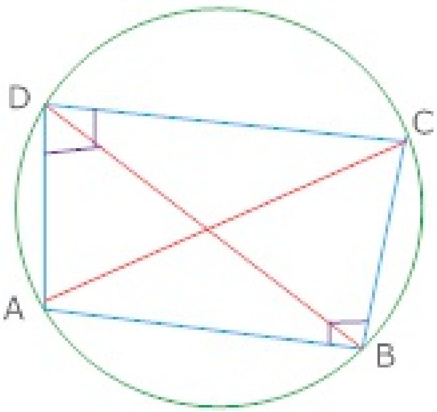
AB = DC

$\angle ABP = \angle DCP$ [Angle in the same segment

$\angle PAB = \angle PDC$ [Angle in the same segment

$\Delta PAB \cong \Delta PDC$ [ASA criterion]

12. Prove that $\angle CAD = \angle CBD$, if ABC and ADC are two right triangle with common hypotenuse AC.



Ans. $\angle ADC = \angle ABC = 90°$ [AC is the common hypotenuse of it Δs ADC and ABC]

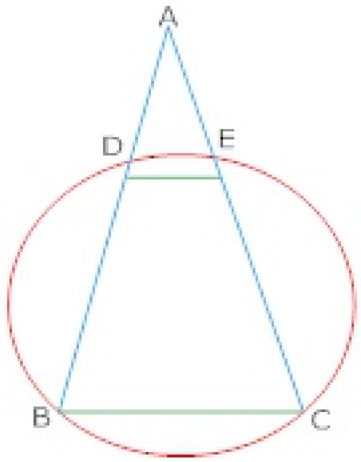
$\angle ADC + \angle ABC = 180°$

$\Rightarrow$Quadrilateral ABCD is cyclic

Now, chord CD subtends $\angle CAD$ and $\angle CBD$

$\angle CAD = \angle CBD$ [Angle in the same segment]

13. Show that $DE \parallel BC$, in isosceles triangle ABC, AB = AC and B,C intersects the sides AB and AC at D and E.



Ans. According to given: BCED forms a cyclic quadrilateral

$\angle AED = \angle B$..........(i)

$\angle C = \angle B$..........(ii)

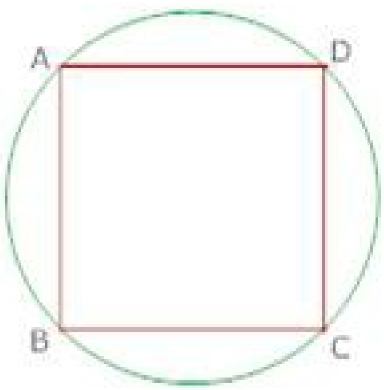
From (i) and (ii) we get

$\angle AED = \angle C$

But they form a Pair of corresponding angles

$DE \parallel BC$

14. Prove that cyclic parallelogram is a rectangle.



Ans. Let ABCD be the given cyclic parallelogram

$\angle A + \angle C = 180°$...........$(i)$

$\angle A = \angle C$

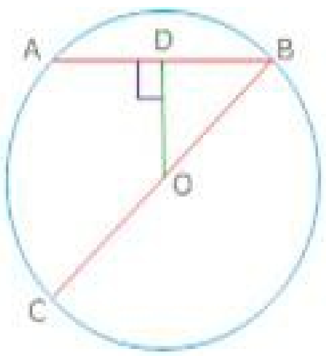
[Opposite angle of a parallelogram are equal]........(ii)

From (i) and (ii)

$\angle A = \angle C = 90^\circ$

$\angle ABCD$ is a rectangle.

15. A line is Passing through the centre of a circle. If it bisects chord AB and CD of the circle. Prove that AB||CD.



Ans. Line EF passes through the centre O and bisects chord AB at P and chord CD at Q

$\therefore$ P, is the mid-Point of AB and Q is the mid-point of CD

But the line joining the mid-point of a chord to the centre of the circle is perpendicular to the chord.

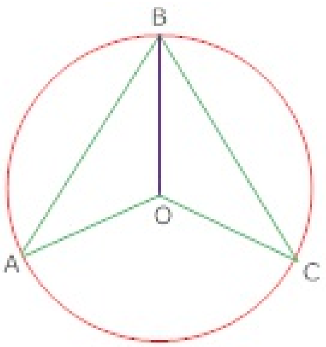
$OP \perp AB$ And $OQ \perp CD$

$\angle OPB = \angle OQD = 90^\circ$

$\angle OPB + \angle OQD = 180^\circ$

$AB \parallel CD$

16. AB and CB are two chords of circle. Prove that BO bisects $\angle ABC$.



Ans. Join OA and OC

In ΔOAB And $\Delta\ OCB$

OA=OC (redii of circle)

OB=OB (common)

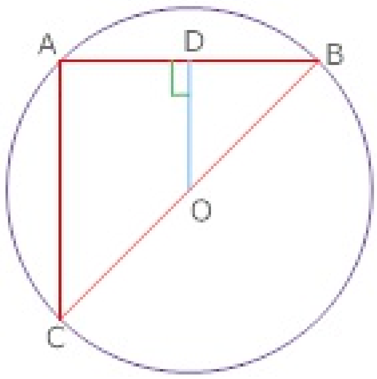
AB=AB (given)

$\Delta OAB = \Delta OCB$ (by SSS)

$\angle ABO = \angle CBO$

Hence, BO bisects $\angle ABC$

17. If BC is diameter of circle with centre O and OD is $\perp$ to chord AB so prove CA=2 OD



Ans. Join AC

Given that $OD \perp AB$

D is the mid-point of AB

O is the mid-Point of BC

Now in ΔABC,

OD is the line joining the mid points of sides BC and AB

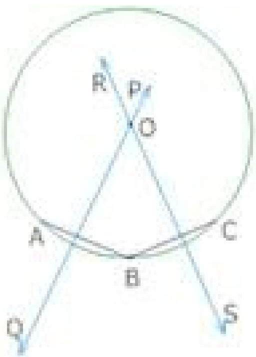
$$OD = \frac{1}{2} AC$$

$AC = 2OD$

$CA = 2OD$

Hence proved.

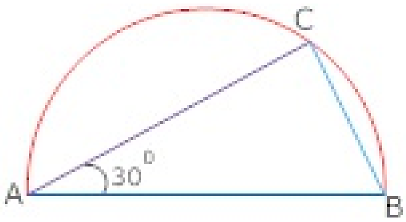
18. Given a method to find the centre of a circle



Ans. Take three distinct points (non-collinear) A, B and C on the circle. Join AB and BC Draw $\perp$ bisectors PQ and RS of AB and BC respectively, to intersect at O

Now, P, is the centre of the circle.

19. C point is taken so that $m\angle CAB = 30^0$ from a semi-circle with AB as diameter. So find $m\angle ACB$ and $m\angle ABC$.



Ans. AB is a diameter and C is a point on the semi-circle

$m\angle ACB = 90°$

$m\angle CAB = 30°$

In ΔACB,

$m\angle ABC + 30° + 90° = 180°$

$m\angle ABC + 120° = 180°$

$m\angle ABC = 180° - 120° = 60°$

$m\angle ACB = 90°$ and $m\angle ABC = 60°$

20. Two different circle can't interact each other at more than two points so, prove it.

Ans. Let the two different circles intersect in three point A, B, C. Then these points A B and C one non-collinear

We know that through three non-collinear Points, one and only one circle can pass so, it contradicts the hypothesis.

21. O is the centre and $OP \perp AB$, find the length of the chord AB

Ans. Perpendicular drawn from the centre to the chord bisects the chord.

$$AP = PB = \frac{1}{2}AB$$

In right angled triangle BPO,

$$OB^2 = OP^2 + BP^2$$

$$(5)^2 = (3)^2 + (BP)^2$$

$$\left(BP = \sqrt{16} = 4cm\right)$$

$$AB = 2BP = 2 \times 4 = 8cm$$

22. If OA is perpendicular to CB, find the length of AB

Ans. $OA \perp CB$

In right angles triangle OAB,

$$OB^2 = OA^2 + AB^2$$

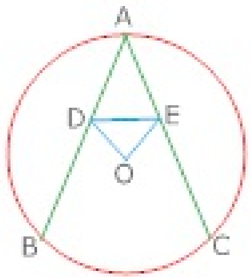
$$OB^2 - OA^2 = AB^2$$

$25-9=AB^2$

$AB^2=16$

AB=4

23. Prove that ADE is an isosceles triangle if $OD \perp AB$ and $OE \perp AC$



Ans. Given that AB and AC is two equal chords of the circle with centre O,

$OD \perp AB$ And $OE \perp AC$

$OD = OE$ [Equal chords are equidistant]

$\angle ODE = \angle OED..........(i)$

$\angle ODA = \angle OEA..........(ii)$

Subtracting (i) from (ii)

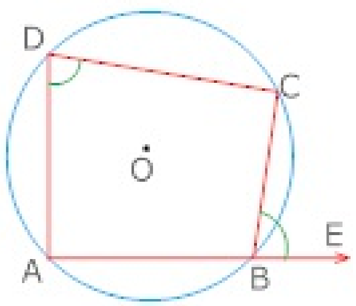
$\angle ODA - \angle OD = \angle OEA - \angle OED$

$\angle ADE = \angle AED$

AD=AE

$\Rightarrow ADE$ is an isosceles Δ

24. Prove that the exterior angle formed by producing a side of a cyclic quadrilateral is equal to the interior opposite angle. Prove



Ans. $\angle ABC + \angle ADC = 180°$ [Opposite angles of a cyclic quadrilateral]

$\angle ABC + \angle CBE = 180°$

$\angle ABC + \angle ADC = \angle ABC + \angle CBE$

$\angle ADC = \angle CBE$

$\angle CBE = \angle ADC$

25. Show that $\angle OMN = \angle ONM$ if AB and CD are two equal chord.

Ans. We know that equal chords of a circle are equidistant from the centre

$AB = CD \Rightarrow OM = ON$

In ΔOMN,

$OM = ON$

$\angle OMN = \angle ONM$

26. From the above question. Show that $\angle BMN = \angle DNM$

Ans. $OM \perp AB$ And $ON \perp CD$

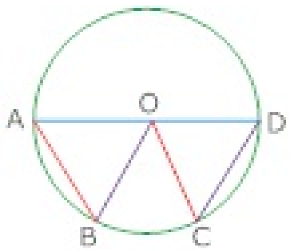
$\angle OMD = \angle OND = 90°$

Adding equal to equals we get

$\angle OMN + \angle OMB = \angle ONM + \angle OND$

$\angle BMN = \angle DNM$

27. Show that $\angle AOB = \angle COD$ if OA and OC are radii of same circle. OB and OD are also radii of same circle.



Ans. In $\triangle$AOB and $\triangle$COD

OA=OC (radii of same circle)

OB=OD (radii of same circle)

AB=CD (given)

$\triangle$AOB $\cong$ $\triangle$COD (by SSS)

$\angle AOB = \angle COD$ [CPCT]

28. Prove that OM Bisect AB. If OM $\perp$ AB.

Ans. AB is a chord of the circle with centre O.

OM $\perp$ AB

OA=OB (radii of same circle)

OM=OM (common)

$\triangle$AOB $\cong$ $\triangle$COD (by SSS)

$\angle OMA = \angle OMB$ [each 90°]

$\triangle$OAM $\cong$ $\triangle$OBM (by SAS)

AB = BM

Hence OM bisects AB

29. Prove that OM $\perp$ AB if AB is chord of the circle with centre O. O is joined to the mid-point M and AB.

Ans. O is joined to the mid-point M to AB

OM=OM (common)

AM=MB (M is midpoint of AM)

OA=OB (radii of same circle)

$\triangle$AOB $\cong$ $\triangle$COD (by SSS)

$\angle$OMA= $\angle$OMB (CPCT)

But $\angle$OMA+ $\angle$OMB =180° (linear pair)

$\because \angle OMA = \angle OMB)$

$\therefore \angle OMA = \dfrac{180^\circ}{2} = 90^\circ$

Thus, $\angle$OMA= $\angle$OMB =90°

Hence, OM $\perp$ AB

30. ABCD is a cyclic quadrilateral in a circle with centre O. Prove that $\angle A + \angle C = 180^\circ$

Ans. If we join OD and OB, we can get $\angle$DOB=2$\angle$C, $\angle$DOB=2$\angle$A

2($\angle$A+$\angle$C) = $\angle$DOB+ reflex $\angle$DOB = 360°

$\angle$A+$\angle$C = 180°

CBSE Class 9 Mathemaics

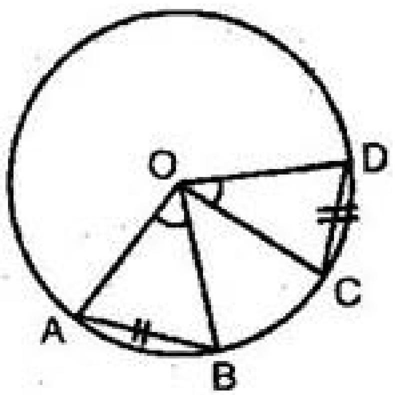
Important Questions

Chapter 10

Circles

3 Marks Questions

1. Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.



Ans. I Part: Two circles are said to be congruent if and only if one of them can be superposed on the other so as to cover it exactly.

Let C (O, r) and C (O', s) be two circles. Let us imagine that the circle C (O', s) is superposed on C (O, r) so that O' coincide with O. Then it can easily be seen that C (O', s) will cover C (O, r) completely if and only if

Hence we can say that two circles are congruent, if and only if they have equal radii.

II Part: Given: In a circle (O, r), AB and CD are two equal chords, subtend $\angle$AOB and $\angle$COB at the centre.

To Prove: $\angle$AOB = $\angle$COD

Proof: In $\triangle$AOB and $\triangle$COD,

AB = CD [Given]

AO = CO [Radii of the same circle]

BO = DO [Radii of the same circle]

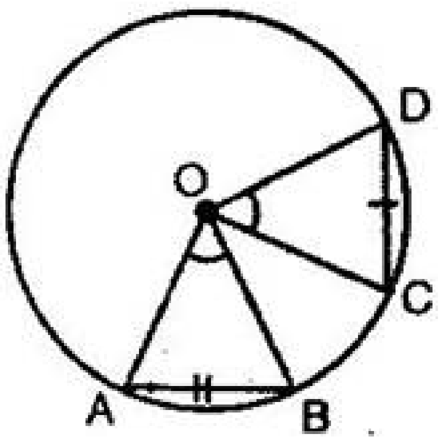
∴ △AOB ≅ △ COD[By SSS axiom]

⇒ ∠AOB = ∠COD[By CPCT]

Hence Proved.

2. Prove that if chords of congruent circles subtend equal angles at their centres, then the chords are equal

Ans. Given: In a circle (O, r), AB and CD subtend two angles at the centre such that ∠AOB = ∠COD



To Prove: AB = CD

Proof: : In △AOB and △COD,

AO = CO[Radii of the same circle]

BO = DO[Radii of the same circle]

∠AOB = ∠COD[Given]

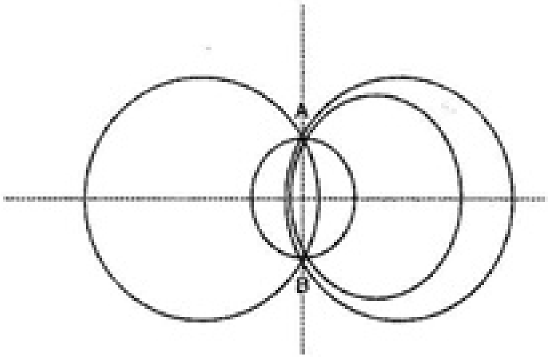
∴ △AOB ≅ △ COD[By SAS axiom]

⇒ AB = CD [By CPCT]

Hence proved.

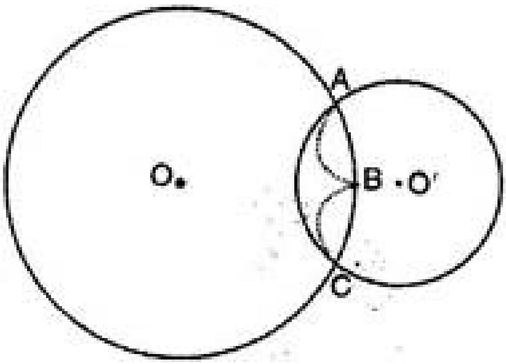
3. Draw different pairs of circles. How many points does each pair have in common?

What is the maximum number of common points?



Ans. From the figure, we observe that when different pairs of circles are drawn, each pair have two points (say A and B) in common.

Maximum number of common points are two in number.



Suppose two circles C (O, r) and C (O', s) intersect each other in three points, say A, B and C.

Then A, B and C are non-collinear points.

We know that:

There is one and only one circle passing through three non-collinear points.

Therefore, a unique circle passes through A, B and C.

$\Rightarrow$ O' coincides with O and $s = r$.

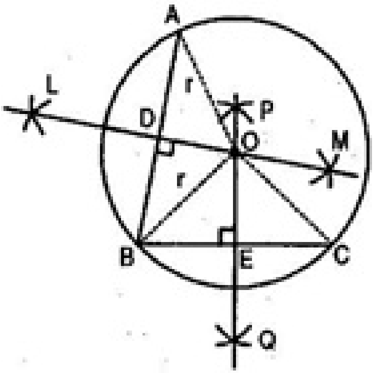
A contradiction to the fact that C (O', s) $\neq$ C (O, r)

$\therefore$ Our supposition is wrong.

Hence two different circles cannot intersect each other at more than two points.

4. Suppose you are given a circle. Give a construction to find its centre.

Ans. Steps of construction:



(a) Take any three points A, B and C on the circle.

(b) Join AB and BC.

(c) Draw perpendicular bisector say LM of AB.

(d) Draw perpendicular bisector PQ of BC.

(e) Let LM and PQ intersect at the point O.

Then O is the centre of the circle.

Verification:

O lies on the perpendicular bisector of AB.

$\therefore$ OA = OB..........(i)

O lies on the perpendicular bisector of BC.

$\therefore$ OB = OC..........(ii)

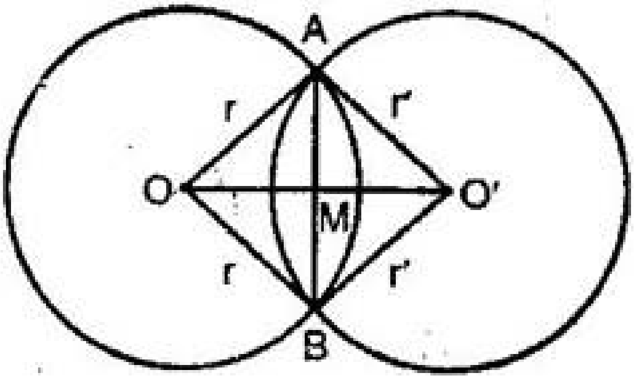
From eq. (i) and (ii), we observe that

OA = OB = OC = r (say)

Three non-collinear points A, B and C are at equal distance (r) from the point O inside the circle.

Hence O is the centre of the circle.

5. If two circles intersect at two points, prove that their centres lie on the perpendicular bisector of the common chord.



Ans. Given: Let C (O, r) and C (O', r') be two circles intersecting at A and B. AB is the common chord.

To prove: OO' is the perpendicular bisector of the chord AB.

Construction: Join OA, OB, O'A, O'B.

Proof: In triangles OAO' and OBO',

OA = OB [Each radius]

O'A = O'B [Each radius]

OO' = OO' [Common]

$\therefore$ $\triangle$ OAO' $\cong$ $\triangle$ OBO'[By SSS congruency]

$\Rightarrow$ $\angle$ AOO' = $\angle$ BOO'[By CPCT]

$\Rightarrow$ $\angle$ AOM = $\angle$ BOM

Now in $\triangle$ AOB,OA = OB

And $\angle$ AOB = $\angle$ OBA [Proved earlier]

Also $\angle$ AOM = $\angle$ BOM

$\therefore$ Remaining $\angle$ AMO = $\angle$ BMO

$\Rightarrow$ $\angle$ AMO = $\angle$ BMO = 90° [Linear pair]

$\Rightarrow$ OM $\perp$ AB

$\Rightarrow$ OO' $\perp$ AB

Since OM $\perp$ AB

$\therefore$ M is the mid-point of AB.

Hence OO' is the perpendicular bisector of AB.

6. Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centers is 4 cm. Find the length of the common chord.

Ans. Let two circles with centres O and O' intersect each other at points A and B. On joining A and B, AB is a common chord.

Radius OA = 5 cm, Radius O'A = 3 cm,

Distance between their centers OO' = 4 cm

In triangle AOO',

$5^2 = 4^2 + 3^2$

$\Rightarrow$ 25 = 16 + 9

$\Rightarrow$ 25 = 25

Hence AOO' is a right triangle, right angled at O'.

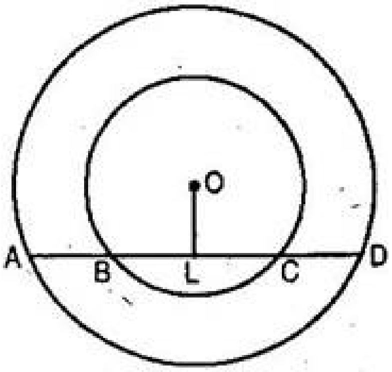
Since, perpendicular drawn from the center of the circle bisects the chord.

Hence O' is the mid-point of the chord AB. Also O' is the centre of the circle II.

Therefore, length of chord AB = Diameter of circle II

$\therefore$ Length of chord $AB = 2 \times 3$ = 6 cm.

7. If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD. (See figure)



Ans. Given: Line l intersects two concentric circles with centre O at points A, B, C and D.

To prove: AB = CD

Construction: Draw OL $\perp l$

Proof: AD is a chord of outer circle and OL $\perp$ AD.

$\therefore$ AL = LD ....(i) [Perpendicular drawn from the centre bisects the chord]

Now, BC is a chord of inner circle and OL $\perp$ BC

$\therefore$ BL = LC ......(ii) [Perpendicular drawn from the centre bisects the chord]

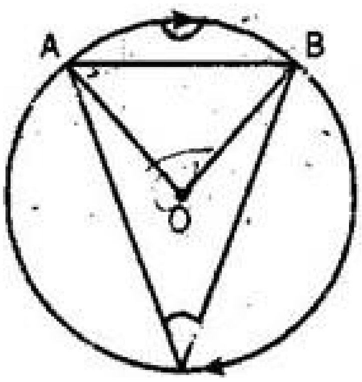
Subtracting (ii) from (i), we get,

AL – BL = LD – LC

$\Rightarrow$ AB = CD

8. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord on a point on the minor arc and also at a point on the major arc.

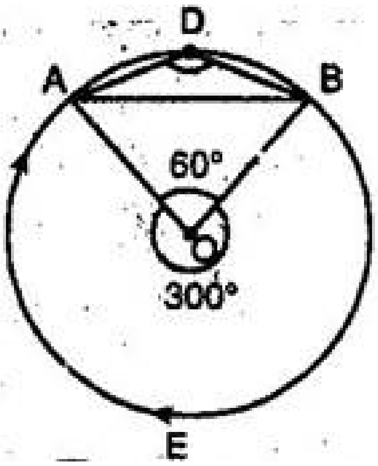
Ans. Let AB be the minor arc of circle.



$\therefore$ Chord AB = Radius OA = Radius OB

$\therefore$ $\triangle$ AOB is an equilateral triangle.

$\Rightarrow$ $\angle$ AOB = 60°



Now $m\widehat{AB} + m\widehat{BA} = 360^\circ$

$\Rightarrow$ $\angle$ AOB + $\angle$ BOA = 360°

$\Rightarrow$ 60° + $\angle$ BOA = 360°

$\Rightarrow$ $\angle$ BOA = $360^\circ - 60^\circ = 300^\circ$

D is a point in the minor arc.

$\therefore$ $m\widehat{BA}$ = 2 $\angle$ BDA

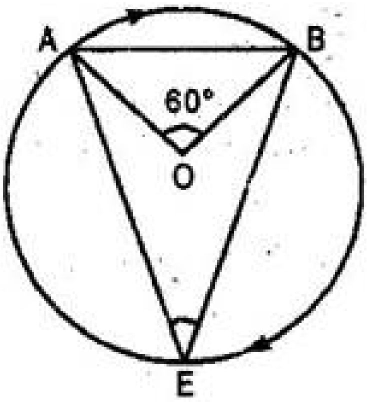
$\Rightarrow$ $\angle$ BOA = 2 $\angle$ BDA

$\Rightarrow$ $\angle$ BDA = $\frac{1}{2}$ $\angle$ BOA = $\frac{1}{2} \times 300^\circ$

$\Rightarrow$ $\angle$ BDA = 150°

Thus angle subtended by major arc, $\widehat{BA}$ at any point D in the minor arc is 150°.

Let E be a point in the major arc $\widehat{BA}$.



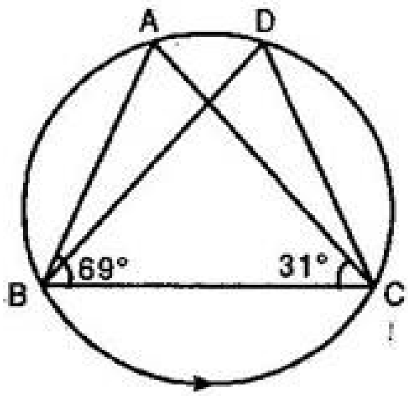
$\therefore m\widehat{AB} = 2\angle AEB$

$\Rightarrow \angle AOB = 2\angle AEB$

$\Rightarrow \angle AEB = \frac{1}{2}\angle AOB$

$\Rightarrow \angle AEB = \frac{1}{2}\times 60^\circ = 30^\circ$

9. In figure, $\angle$ABC = 69°, $\angle$ACB = 31°, find $\angle$BDC.



Ans. In triangle ABC,

$\angle BAC + \angle ABC + \angle ACB = 180^\circ$

$\Rightarrow \angle BAC + 69^\circ + 31^\circ = 180^\circ$

$\Rightarrow \angle BAC = 180^\circ - 69^\circ - 31^\circ$

$\Rightarrow \angle BAC = 80^\circ$ ....(i)

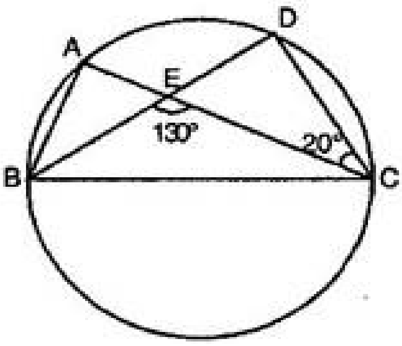
Since, A and D are the points in the same segment of the circle.

$\therefore$ $\angle$BDC = $\angle$BAC

[Angles subtended by the same arc at any points in the alternate segment of a circle are equal]

$\Rightarrow$ $\angle$BDC = 80° [Using (i)]

10. In figure, A, B, C, D are four points on a circle. AC and BD intersect at a point E such that $\angle$BEC = 130° and $\angle$ECD = 20°. Find $\angle$BAC.



Ans. Given: $\angle$BEC = 130° and $\angle$ECD = 20°

$\angle$DEC = $180^\circ - \angle$ BEC = $180^\circ - 130^\circ = 50^\circ$ [Linear pair]

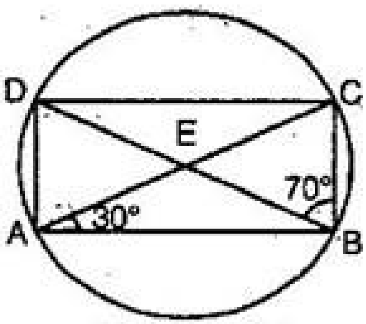
Now in ΔDEC,

$\angle$DEC + $\angle$DCE + $\angle$EDC = 180° [Angle sum property]

$\Rightarrow$ $50^\circ + 20^\circ + \angle$EDC = 180° $\Rightarrow$ $\angle$EDC = 110°

$\Rightarrow$ $\angle$BAC = $\angle$EDC = 110° [Angles in same segment]

11. If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.



Ans. Since AC is a diameter.

$\therefore \angle B = \angle D = 90^\circ$ ....(i)

[Angle in semicircle is right angle]

Similarly $\angle A = \angle C = 90^\circ$ ....(ii)

Now AC = BD[Diameters of same circle]

$\Rightarrow \overset{\frown}{AC} \cong \overset{\frown}{BD}$ [Arcs opposite to equal chords]

$\Rightarrow \overset{\frown}{AC} - \overset{\frown}{DC} \cong \overset{\frown}{BD} - \overset{\frown}{DC}$

$\Rightarrow \overset{\frown}{AD} \cong \overset{\frown}{BC}$

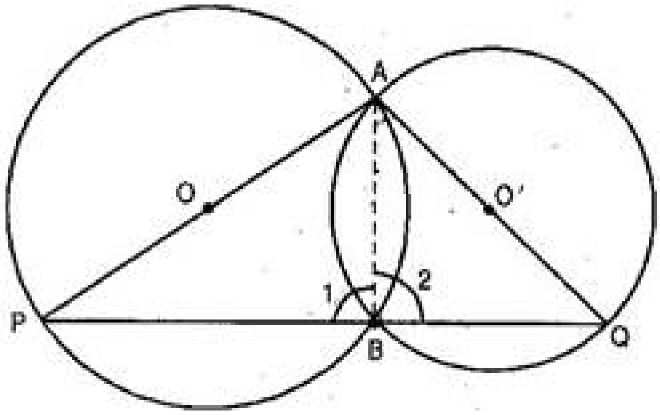
$\Rightarrow$ AD = BC[Chords opposite to equal arcs] ....(iii)

Similarly AB = DC..........(iv)

From eq. (i), (ii), (iii) and (iv), we observe that each angles of the quadrilateral is 90° and opposite sides are equal.

Hence ABCD is a rectangle.

12. If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.



Ans. Given: Two circles intersect each other at points A and B. AP and AQ be their respective diameters.

To prove: Point B lies on the third side PQ.

Construction: Join A and B.

Proof: AP is a diameter.

$\therefore \angle 1 = 90^\circ$

[Angle in semicircle]

Also AQ is a diameter.

$\therefore \angle 2 = 90^\circ$

[Angle in semicircle]

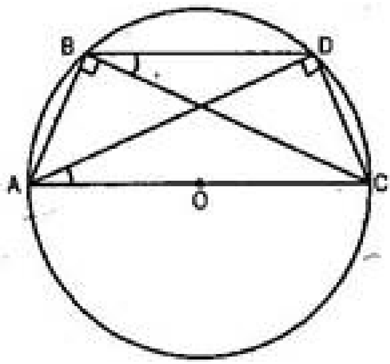
$\angle 1 + \angle 2 = 90^\circ + 90^\circ$

$\Rightarrow \angle PBQ = 180^\circ$

$\Rightarrow$ PBQ is a line.

Thus point B. i.e. point of intersection of these circles lies on the third side i.e., on PQ.

13. ABC and ADC are two right triangles with common hypotenuse AC. Prove that $\angle$ CAD = $\angle$ ABD.



Ans. We have ABC and ADC two right triangles, right angled at B and D respectively.

$\Rightarrow \angle$ ABC = ADC[Each 90°]

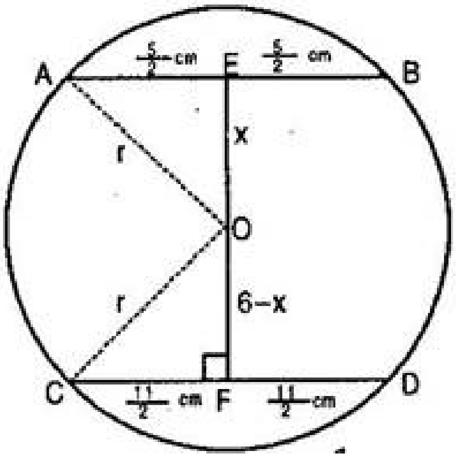
If we draw a circle with AC (the common hypotenuse) as diameter, this circle will definitely passes through of an arc AC, Because B and D are the points in the alternate segment of an arc AC.

Now we have $\overset{\frown}{CD}$ subtending $\angle$CBD and $\angle$CAD in the same segment.

$\therefore$ $\angle$CAD = $\angle$CBD

Hence proved.

14. Two chords AB and CD of lengths 5 cm and 11 cm respectively of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD is 6 cm, find he radius of the circle.



Ans. Let O be the centre of the circle.

Join OA and OC.

Since perpendicular from the centre of the circle to the chord bisects the chord.

$\therefore$ AE = EB = $\frac{1}{2}$ AB = $\frac{1}{2}\times 5$ = $\frac{5}{2}$ cm

And CF = FD = $\frac{1}{2}$ CD = $\frac{1}{2}\times 11$ = $\frac{11}{2}$ cm

Let OE = x

$\therefore$ OF = $6-x$

Let radius of the circle be r.

In right angled triangle AEO,

$AO^2 = AE^2 + OE^2$

[Using Pythagoras theorem]

$\Rightarrow r^2 = \left(\frac{5}{2}\right)^2 + x^2$ ....(i)

Again In right angled triangle CFO,

$OC^2 = CF^2 + OF^2$

[Using Pythagoras theorem]

$\Rightarrow r^2 = \left(\frac{11}{2}\right)^2 + (6-x)^2$ ....(ii)

Equating eq. (i) and (ii),

$\left(\frac{5}{2}\right)^2 + x^2 = \left(\frac{11}{2}\right)^2 + (6-x)^2$

$\Rightarrow \frac{25}{4} + x^2 = \frac{121}{4} + 36 + x^2 - 12x$

$\Rightarrow 12x = \frac{121}{4} - \frac{25}{4} + 36$

$\Rightarrow 12x = \frac{96}{4} + 36$

$\Rightarrow 12x = 24 + 36$

$\Rightarrow 12x = 60$

$\Rightarrow x = 5$

Now from eq. (i),

$$r^2 = \frac{25}{4} + x^2$$

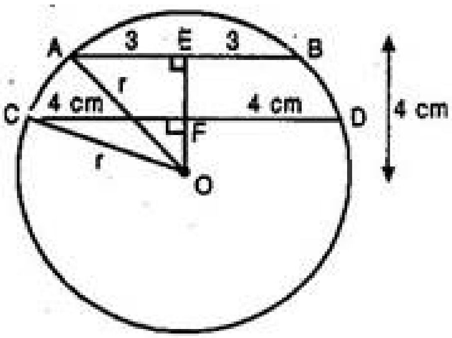
$$\Rightarrow r^2 = \frac{25}{4} + 5^2$$

$$\Rightarrow r^2 = \frac{125}{4}$$

$$\Rightarrow r = \frac{5\sqrt{5}}{2} \text{ cm}$$

Hence radius of the circle is $\frac{5\sqrt{5}}{2}$ cm.

15. The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at a distance of 4 cm from the centre, what is the distance of the other chord form the centre?



Ans. Let AB = 6 cm and CD = 8 cm are the chords of circle with centre O.

Join OA and OC.

Since perpendicular from the centre of the circle to the chord bisects the chord.

$\therefore$ AE = EB = $\frac{1}{2}$ AB = $\frac{1}{2} \times 6$ = 3 cm

And CF = FD = $\frac{1}{2}$ CD = $\frac{1}{2} \times 8$ = 4 cm

Perpendicular distance of chord AB from the centre O is OE.

$\therefore$ OE = 4 cm

Now in right angled triangle AOE,

$OA^2 = AE^2 + OE^2$ [Using Pythagoras theorem]

$\Rightarrow r^2 = 3^2 + 4^2$

$\Rightarrow r^2 = 9 + 16 = 25$

$\Rightarrow r = 5$ cm

Perpendicular distance of chord CD from the center O is OF.

Now in right angled triangle OFC,

$OC^2 = CF^2 + OF^2$ [Using Pythagoras theorem]

$\Rightarrow r^2 = 4^2 + OF^2$

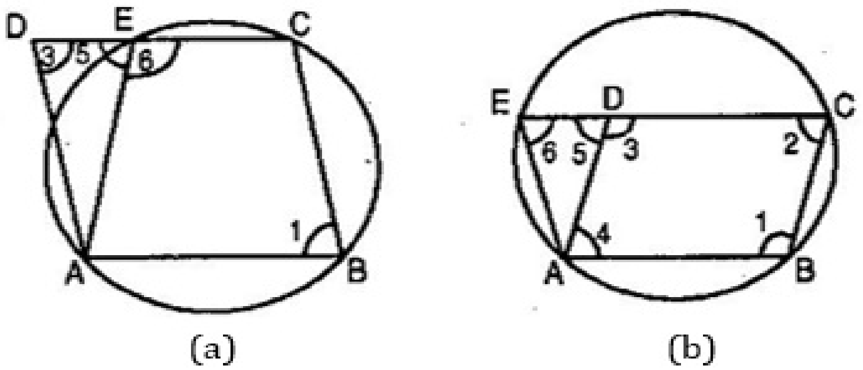
$\Rightarrow 5^2 = 16 + OF^2$

$\Rightarrow OF^2 = 16$

$\Rightarrow$ OF = 3cm

Hence distance of other chord from the centre is 3 cm.

16. ABCD is a parallelogram. The circle through A, B and C intersect CD (produced it necessary) at E. Prove that AE = AD.



Ans. In figure (a),

ABCD is a parallelogram.

$\Rightarrow \angle 1 = \angle 3$..........(i)

ABCE is a cyclic quadrilateral.

$\therefore \angle 1 + \angle 6 = 180^\circ$ ....(ii)

And $\angle 5 + \angle 6 = 180^\circ$..........(iii) [Linear pair]

From eq. (ii) and (iii), $\angle 1 = \angle 5$..........(iv)

Now, from eq. (i) and (iv),

$\angle 3 = \angle 5 \Rightarrow$ AE = AD [Sides opposite to equal angles are equal]

In figure (b),

ABCD is a parallelogram.

$\therefore \angle 1 = \angle 3$ and $\angle 2 = \angle 4$

Also AB $\parallel$ CD and BC meets them.

$\therefore \angle 1 + \angle 2 = 180^\circ$ ....(i)

And AD $\parallel$ BC and EC meets them.

$\therefore \angle 5 = \angle 2$ ....(ii) [Corresponding angles]

Since ABCE is a cyclic quadrilateral.

$\therefore \angle 1 + \angle 6 = 180^\circ$ ....(iii)

From eq. (i) and (iii),

$\angle 1 + \angle 2 = \angle 1 + \angle 6$

$\Rightarrow \angle 2 = \angle 6$

But from eq. (ii), $\angle 2 = \angle 5$

$\therefore \angle 5 = \angle 6$

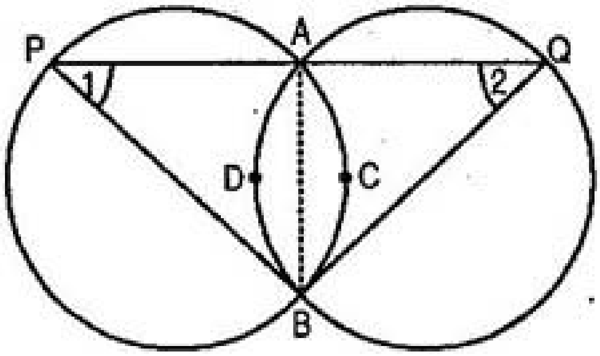
Now in triangle AED,

$\angle 5 = \angle 6$

$\Rightarrow$ AE = AD [Sides opposite to equal angles]

Hence in both the cases, AE = AD

17. Two congruent circles intersect each other at points A and B. Through A any line segment PAQ is drawn so that P, Q lie on the two circles. Prove that BP = BQ.



Ans. Given: Two equal circles intersect in A and B.

A straight line through A meets the circles in P and Q.

To prove: BP = BQ

Construction: Join A and B.

Proof: AB is a common chord and the circles are equal.

$\therefore$ Arc about the common chord are equal, i.e.,

$\overset{\frown}{ACB} = \overset{\frown}{ADB}$

Since equal arcs of two equal circles subtend equal angles at any point on the remaining part of the circle, then we have,

$\angle 1 = \angle 2$

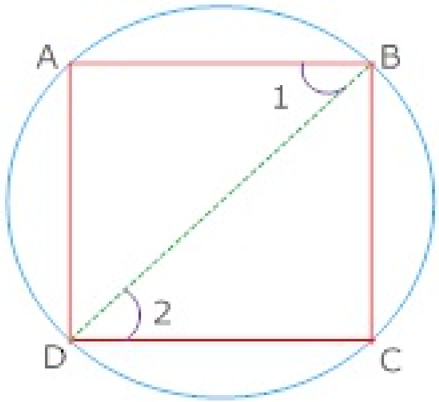
In triangle PBQ,

$\angle 1 = \angle 2$ [proved]

$\therefore$ Sides opposite to equal angles of a triangle are equal.

Then we have, BP = BQ

18. Pair of opposite sides of a cyclic quadrilateral are equal, Prove that the other two sides are parallel.



Ans. Given: A cyclic quadrilateral ABCD in which AD= BC

To Prove: AB | | CD

Construction: Join B and D

Proof: AD=BC

$\widehat{AD} = \widehat{BC}$

$\angle 1 = \angle 2$

But these are alternate angles

AB | | CD

19. Prove that the centre of the circle through A, B, C, D is the Point intersection of its diagonals.

Ans. Given: A cyclic rectangle ABCD in which diagonals AC and BD intersect at Point O

To Prove: O is the centre of the circle

Proof: ABCD is a rectangle

AC= BD

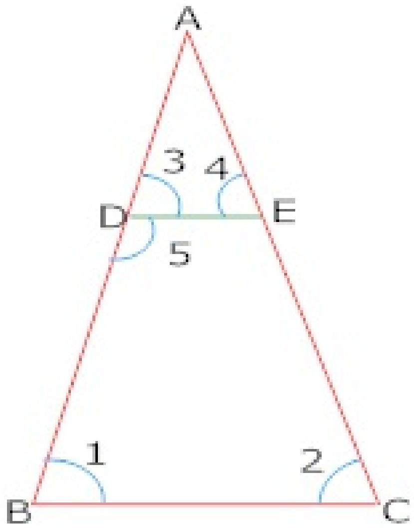
Now as the diagonals AC and BD are intersecting at O

AO=OC, OB=OD

AO=OC=OB=OD

A, B, C, D lie on the same circle.

20. In isosceles triangle ABC, AD = AE and D and E are equal on side AB and AC so prove that B,C,E and Δ are cyclic



Ans. Given that $\Delta ABC, AB = AC$

AD=AE

$$\frac{AD}{AB} = \frac{AE}{AC}$$

$DE \parallel BC$

$\angle 3 = \angle 1, \angle 4 = \angle 2$

$\angle 4 = \angle 3$

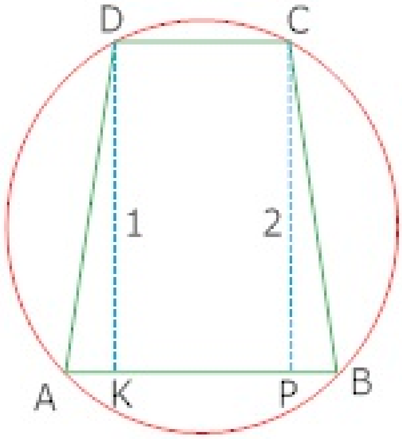
$\angle 3 = \angle 2$

$\angle 3 + \angle 5 = \angle 2 + \angle 5$ [Adding $\angle 5$ both side]

$\angle 3 + \angle 5 = 180°$

$\angle 2 + \angle 5 = 180°$

21. If two non – parallel sides of a trapezium are equal, prove that it is cyclic.



Ans. In it ΔADK and $\Delta\ BCP$

AD=BC

DK=CP [Distance between | | sides

$\Delta ADK = \Delta BCP$

$\angle A = \angle B$..........(i)

$\angle 1 = \angle 2$

$\angle 1 + 90° = \angle 2 + 90°$

$\angle ADC = \angle BCD$

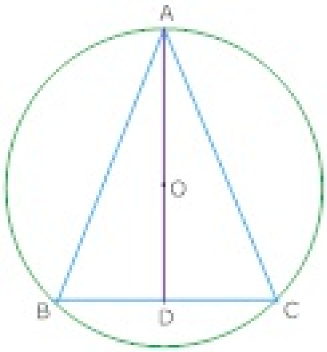
$\angle D = \angle C$..............(ii)

$\angle A + \angle B + \angle C + \angle D = 360°$

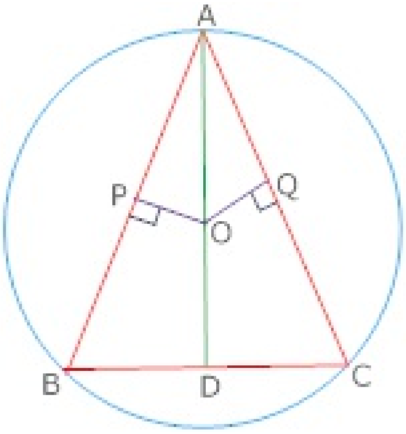
$\angle B + \angle B + \angle D + \angle D = 360°$

$\angle B + \angle D = 180°$

22. In circle bisector AD of $\angle BAC$ of ΔABC Passes through the center O of the circum circle of ΔABC Prove AB=AC



Ans. Draw $OP \perp AB$ and $OQ \perp AC$



In ΔOPA and ΔOQA

$\angle PAO = \angle QAO$

$\angle APO = \angle AQO$

AO=AO

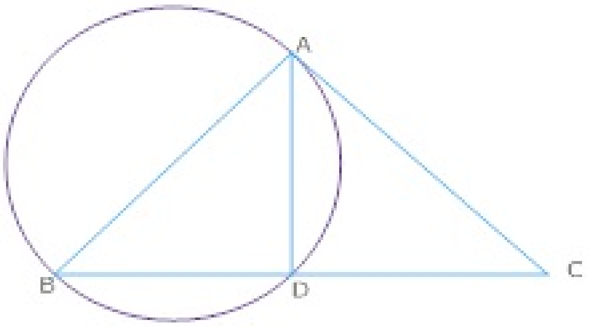
$\Delta OPA \cong \Delta OQA$

OP=OQ

Chords AB and AC are equidistant from centre O

AB=AC

23. Prove that the circle drawn with the equal sides as a diameter passes through the Point D. if D is the mid Point of BC of an isosceles triangle ABC with AB=AC



Ans. Join AD in Δ ABD and Δ ACD

AB=AC

AD=AD

BD=CD [D is mid points]

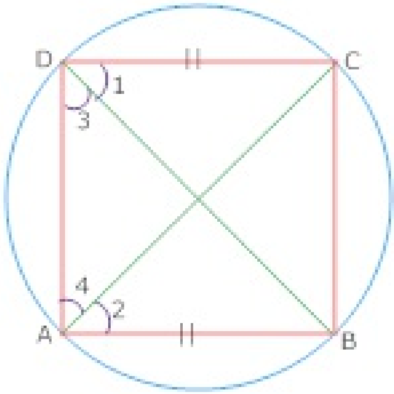
$\Delta ABD \cong \Delta ACD$

$\angle ADB = \angle ADC \quad [CPCT]$

$\angle ADB + \angle ADC = 180°$

$\angle ADB = \angle ADC = 90°$

24. If a Pair of opposite sides of a cyclic quadrilateral are equal, then the diagonals are also equal.



Ans. Given: A cyclic quadrilateral ABCD in which AB=DC

To Prove: diagonal AC=diagonal BD

Proof: $\angle 1 = \angle 2$ (Angle in same segment of circle)

$\angle 3 = \angle 4$

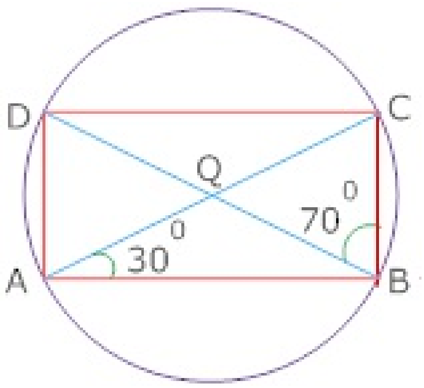
$\angle 1 + \angle 3 = \angle 2 + \angle 4$

$\angle ADC = \angle BAD$

But these are the angles subtended by the diagonals AC and BD in the same circle

AC=BD

25. $\angle DBC = 70°$ **and** $\angle CAB = 30°$ **find** $\angle BCD$



Ans. $\angle DBC = \angle DAC = 70°$ (Angle in same segment)

$\angle DAB = \angle DAC + \angle CAB$

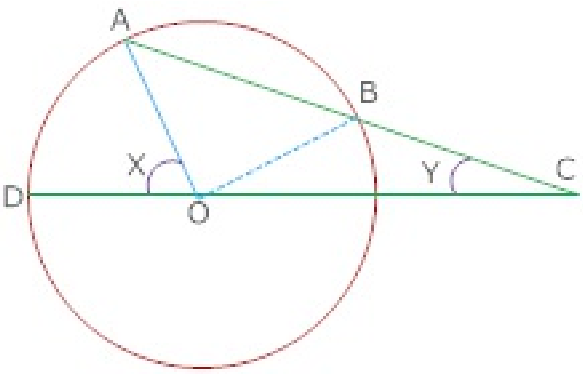
$= 70° + 30° = 100°$

$\angle DAB + \angle BCD = \angle 180°$

$100° + \angle BCD = 180°$

$\angle BCD = 180° - 100° = 80°$

26. AB is chord of a circle and AB Produced to C such that BC=OB and CO joined and produce the circle and meet to D if $\angle ACD = y°$ **and** $\angle AOD = x°$, **prove that** $x = 3y$



Ans. Proof: In ΔBOC,

BO=BC

$\angle BOC = \angle BCO = y$

In ΔBOC, CB is produced to A, forming exterior $\angle OBA$

$\angle OBA = \angle BOC + \angle BCO = y + y = 2y$

OB=OA [Radii of the same circle

$\angle OBA = \angle OAB = 2y$

Again in ΔACO, CO is produced to D, forming exterior $\angle AOD$

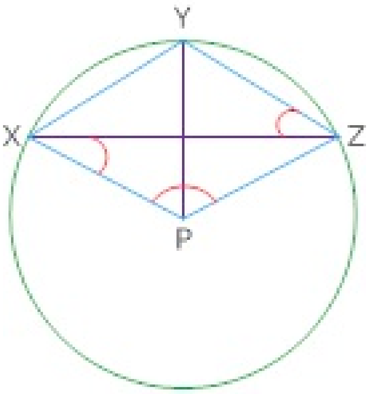
$\angle AOD = \angle OAC + \angle ACO$

$x = 2y + y$

$x = 3y$

Hence proved.

27. Prove that $\angle XPZ = 2(\angle xzy + \angle yxz)$ **if P is the centre of circle**



Ans. Given: A circle with centre P, XY and YZ are two chords

To Prove: $\angle XPZ = 2(\angle XZY + \angle YXZ)$

Proof: $\angle XPY = 2\angle XZY$............(i)

Similarly arc YZ subtends $\angle YPZ$ at centre and $\angle YXZ$ at remaining Part of the circle

$\angle YPZ = 2\angle YXZ.........(ii)$

Adding (i) and (ii) $\angle XPY + \angle YPZ = 2\angle XZY + 2\angle YXZ$

$\angle XPZ = 2(\angle XZY + \angle YXZ)$

28. Prove that OA is the perpendicular bisector of BC if $\widehat{AB} \cong \widehat{AC}$

Ans. Let OA intersect BC in P produce AO to meet the circle at K

Now, AOK is the diameter

$\widehat{ABK} \cong \widehat{ACK}$

$A\widehat{BK} - A\widehat{B} = A\widehat{C}K - A\widehat{C}$

$B\widehat{K} \cong \widehat{CK}$

$\angle 1 = \angle 2$

In ΔABP and ΔACP

AB=AC

AP=AP (Common)

$\angle 1 = \angle 2$

$\Delta ABP \cong \Delta ACP$ (SAS)

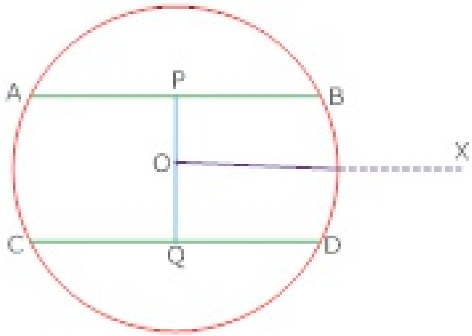
BP=CP

$\angle APB = \angle APC$

$\angle APB + \angle APC = 180°$

Each = $90°$

29. Prove that the line joining the midpoint of the two parallel chords of a circle passes through the centre of the circle.



Ans. Let AB and CD be the two parallel chords of the circle with centre O P and Q are the mid-points of AB and CD join OP and OQ.

Draw $OX \parallel AB$ or CD

$OP \perp AB$ And $OQ \perp CD$

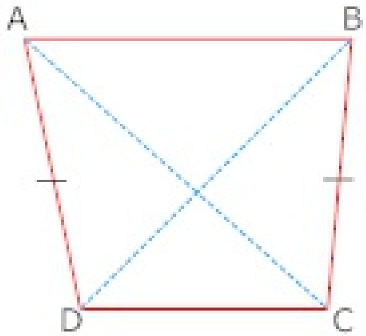
$\angle POX = 90^\circ$ And also $\angle QOX = 90^\circ$

$\angle POX + \angle QOX = 180^\circ$

POQ is a straight line.

30. ABCD is a quadrilateral in which AD=BC and $\angle ADC = \angle BCD$ show A, B, C, D lie on a circle

Ans. Join AC and BD



In ΔACD and ΔBDC

AD=BC

$\angle ADC = \angle BCD$

$DC = DC$ [Common]

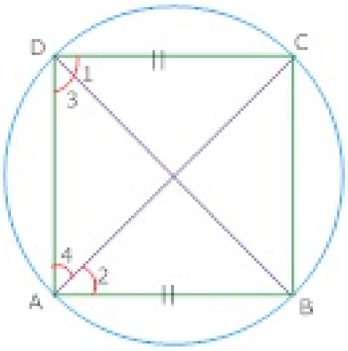
$\Delta ACD \cong \Delta BDC$ [By S.A.S]

$\angle DAC = \angle DBC$

As these are two equal angles on the same side of a line segment CD.

The four points A, B, C and D are concyclic.

31. Prove that diagonal is also equal when pair of opposite sides of a cyclic quadrilateral are equal. Prove.



Ans. Given: A cyclic quadrilateral ABCD in which AB= DC

To Prove: diagonal AC= Diagonal BD

Proof: $\angle 1 = \angle 2$ [Angle in the same segment

$\angle 3 = \angle 4$

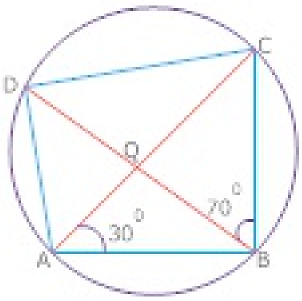
$\angle 1 + \angle 3 = \angle 2 + \angle 4$

$\angle ADC = \angle BAC$

But these are the angle subtended by the diagonal AC and BD in the same circle.

AC=BD

32. In ABCD cyclic quadrilateral diagonal Intersect at $Q.\angle DBC = 70°$ **and** $\angle CAB = 30°$ **so find** $\angle BCD$



Ans. $\angle DBC = \angle DAC = 70°$ [Angle in the same segment]

$\angle DAB = \angle DAC + \angle CAB$

$70° + 30° = 100°$

$\angle DAB + \angle BCD = 180°$

$100° + \angle BCD = 180°$

$\angle BCD = 180° - 100° = 80$

33. Find the value of x if A, B, C, D are concylic points

Ans. $\angle ABC = 180° - 130°$ [Linear pair]

$= 50°$

$\angle x = \angle ABC$ [Exterior angle of a cyclic quadrilateral = interior Opp. angle]

$\angle x = 50°$

34. Calculate the measure of $\angle PQB$, where O is the centre of the circle

Ans. $\angle$ ABP = $42°$

$\angle$ APB = $90°$

$\angle$ APB+$\angle$ ABP+$\angle$ PAB = $180°$

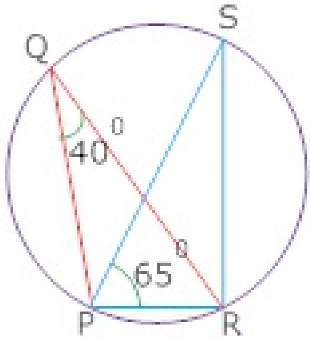
$90° + 42° + \angle$ PAB = $180°$

$\angle$ PAB = $180° - 90° - 42° = 48°$

$\overset{\frown}{PB}$ Subtends ∠ PAB and ∠ PQB in the same segment of the circle

∠ PAB= ∠ PQB=48°

35. In the given Fig $\angle PQR = 40°$ $\angle SPR = 65°$ find $\angle SRP$.



Ans. $\overset{\frown}{PR}$ Subtends ∠ PQR and ∠ PSR in the same segment of the circle

∠ PQR = ∠ PSR = $40°$

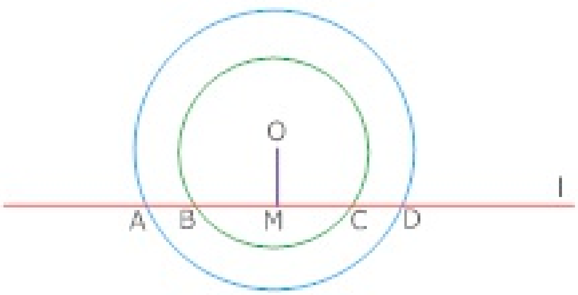
∠ SPR + ∠ PSR + ∠ SRP = $180°$ (angle sum property of a triangle)

$65° + 40° + 180° + \angle SRP = 180°$

∠ SRP = $180° - 40° - 65°$

= $75°$

36. Find the length of AB, CD,AC and BD if two concentric circles with centre O have A,B,C,D as the Point of intersection with line *l*.



Ans. OM ⊥ BC

BM=MC........(i)

OM ⊥ AD

AM = MD.......(ii)

From (i) and (ii)

AM - BM = MD - MC = 6-4

AB = CD = 2cm

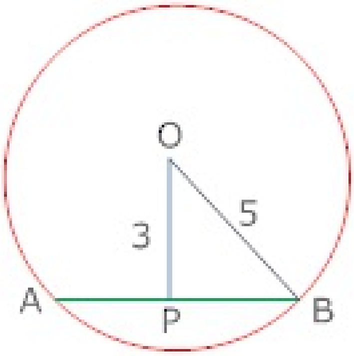
AC = AB+BC

=2+8=10cm

BD=BC+CD

=8+2 = 10cm

37. If OP $\perp$ AB find the length of the chord AB.



Ans. AP = PB = $\frac{1}{2}$ AB

In right angled triangle BPO,

$$OB^2 = OP^2 + BP^2$$

$$(5)^2 = (3)^2 + (BP)^2$$

$$(BP)^2 = 25 - 9 = 16$$

$$BP = \sqrt{16} = 4cm$$

$$AB = 2BP = 2 \times 4 = 8cm$$

CBSE Class 9 Mathemaics

Important Questions

Chapter 10

Circles

4 Marks Questions

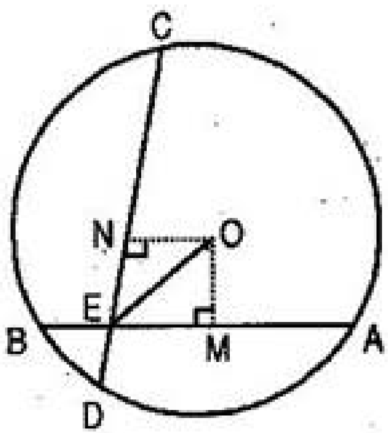
1. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

Ans. Given: Let AB and CD are two equal chords of a circle of centers

O intersecting each other at point E within the circle.

To prove: (a) AE = CE (b) BE = DE

Construction: Draw OM $\perp$ AB, ON $\perp$ CD. Also join OE.



Proof: In right triangles OME and ONE,

$\angle$OME = $\angle$ONE = 90°

OM = ON

[Equal chords are equidistance from the centre]

OE = OE [Common]

$\therefore$ $\triangle$OME $\cong$ $\triangle$ONE [RHS rule of congruency]

$\therefore$ ME = NE [By CPCT] ....(i)

Now, O is the centre of circle and OM $\perp$ AB

$\therefore$ AM = $\frac{1}{2}$ AB [Perpendicular from the centre bisects the chord] .....(ii)

Similarly, NC = $\frac{1}{2}$ CD ....(iii)

But AB = CD [Given]

From eq. (ii) and (iii), AM = NC ....(iv)

Also MB = DN ....(v)

Adding (i) and (iv), we get,

AM + ME = NC + NE

$\Rightarrow$ AE = CE [Proved part (a)]

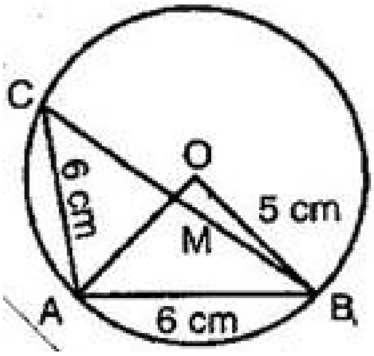
Now AB = CD [Given]

AE = CE [Proved]

$\Rightarrow$ AB - AE = CD - CE

$\Rightarrow$ BE = DE [Proved part (b)]

2. Three girls Reshma, Salma and Mandip are standing on a circle of radius 5 m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6 m each, what is the distance between Reshma and Mandip?



Ans. Let Reshma, Salma and Mandip takes the position C, A and B on the circle.

Since AB = AC

The centre lies on the bisector of $\angle$BAC.

Let M be the point of intersection of BC and OA.

Again, since AB = AC and AM bisects $\angle$CAB.

$\therefore$ AM $\perp$ CB and M is the mid-point of CB.

Let OM = x, then MA = $5-x$

From right angled triangle OMB, $OB^2 = OM^2 + MB^2$

$\Rightarrow$ $5^2 = x^2 + MB^2$ ....(i)

Again, in right angled triangle AMB, $AB^2 = AM^2 + MB^2$

$\Rightarrow$ $6^2 = (5-x)^2 + MB^2$ ....(ii)

Equating the value of MB^2 from eq. (i) and (ii),

$5^2 - x^2 = 6^2 - (5-x)^2$

$\Rightarrow$ $(5-x)^2 - x^2 = 6^2 - 5^2$

$\Rightarrow$ $25 - 10x + x^2 - x^2 = 36 - 25$

$\Rightarrow$ $10x = 25 - 11$

$\Rightarrow 10x = 14 \Rightarrow \quad x = \frac{14}{10}$

Hence, from eq. (i),

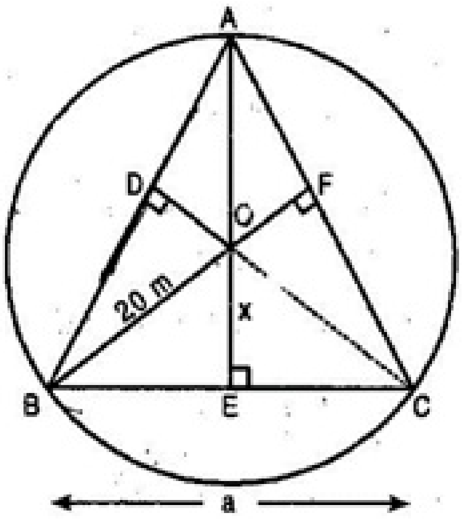
$MB^2 = 5^2 - x^2 = 5^2 - \left(\frac{14}{10}\right)^2$

$= \left(5 + \frac{4}{10}\right)\left(5 - \frac{14}{10}\right) = \frac{64}{10} \times \frac{36}{10}$

$\Rightarrow MB = \frac{8 \times 6}{10} = 4.8$ cm

$\therefore BC = 2MB = 2 \times 4.8 = 9.6\ cm$

3. A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.



Ans. Let position of three boys Ankur, Syed and David are denoted by the points A, B and C respectively.

A = B = C = a [say]

Since equal sides of equilateral triangle are as equal chords and perpendicular distances of equal chords of a circle are equidistant from the centre.

$\therefore$ OD = OE = OF = x cm [say]

Join OA, OB and OC.

$\Rightarrow$ Area of Δ AOB = Area of Δ BOC = Area of Δ AOC

And Area of Δ ABC

= Area of Δ AOB + Area of Δ BOC + Area of Δ AOC

$\Rightarrow$ And Area of Δ ABC = $3 \times Area\ of BOC$

$$\Rightarrow \frac{\sqrt{3}}{4}a^2 = 3\left(\frac{1}{2}BC \times OE\right)$$

$$\Rightarrow \frac{\sqrt{3}}{4}a^2 = 3\left(\frac{1}{2} \times a \times x\right)$$

$$\Rightarrow \frac{a^2}{a} = 3 \times \frac{1}{2} \times \frac{4}{\sqrt{3}} \times x$$

$\Rightarrow a = 2\sqrt{3}x$ ....(i)

Now, CE $\perp$ BC

$\therefore$ BE = EC = $\frac{1}{2}$ BC [$\because$ Perpendicular drawn from the centre bisects the chord]

$\Rightarrow$ BE = EC = $\frac{1}{2}a$

$\Rightarrow$ BE = EC = $\frac{1}{2}\left(2\sqrt{3}x\right)$ [Using eq. (i)]

$\Rightarrow$ BE = EC = $\sqrt{3}x$

Now in right angled triangle BEO,

$OE^2 + BE^2 = OB^2$ [Using Pythagoras theorem]

$\Rightarrow x^2 + \left(\sqrt{3}x\right)^2 = (20)^2$

$\Rightarrow x^2 + 3x^2 = 400$

$\Rightarrow 4x^2 = 400$

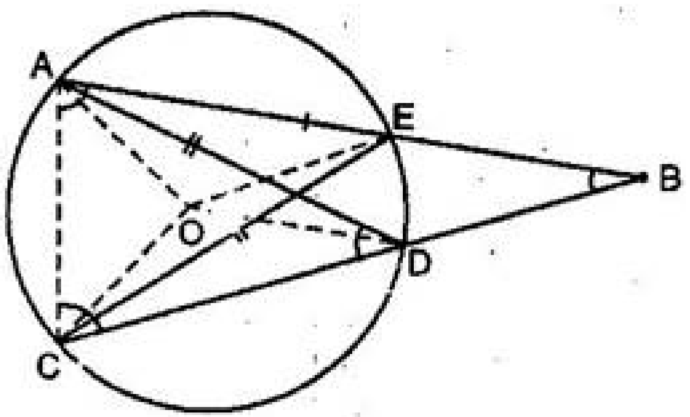
$\Rightarrow x^2 = 100$

$\Rightarrow x = 10$ m

And $a = 2\sqrt{3}x = 2\sqrt{3} \times 10 = 20\sqrt{3}$ m

Thus distance between any two boys is $20\sqrt{3}$ m.

4. Let vertex of an angle ABC be located outside a circle and let the sides of the angle intersect chords AD and CE with the circle. Prove that $\angle$ABC is equal to half the difference of the angles subtended by the chords AC and DE at the centre.



Ans. Vertex B of $\angle$ABC is located outside the circle with centre O.

Side AB intersects chord CE at point E and side BC intersects chord AD at point D with the circle.

We have to prove that

$\angle ABC = \frac{1}{2} [\Delta AOC - \Delta DOE]$

Join OA, OC, OE and OD.

Now $\angle AOC = 2 \angle AEC$

[Angle subtended by an arc at the centre of the circle is twice the angle subtended by the same arc at any point in the alternate segment of the circle]

$\Rightarrow \frac{1}{2} \angle AOC = \angle AEC$ ....(i)

Similarly $\frac{1}{2} \angle DOE = \angle DCE$ ....(ii)

Subtracting eq. (ii) from eq. (i),

$\frac{1}{2} [\Delta AOC - \Delta DOE] = \angle AEC - \angle DCE$ ....(iii)

Now $\angle AEC = \angle ADC$ [Angles in same segment in circle] ....(iv)

Also $\angle DCE = \angle DAE$ [Angles in same segment in circle] ....(v)

Using eq. (iv) and (v) in eq. (iii),

$\frac{1}{2} [\Delta AOC - \Delta DOE] = \angle DAE + \angle ABD - \angle DAE$

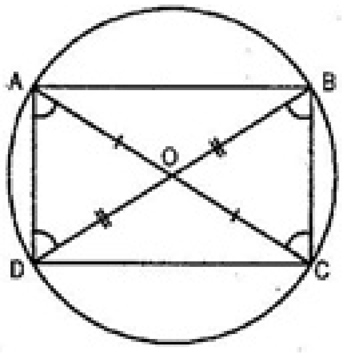
$\Rightarrow \frac{1}{2} [\Delta AOC - \Delta DOE] = \angle ABD$

Or $\frac{1}{2} [\Delta AOC - \Delta DOE] = \angle ABC$ Hence proved.

5. AC and BD are chords of a circle which bisect each other. Prove that:

(i) AC and BD are diameters.

(ii) ABCD is a rectangle.



Ans. Given: AC and BD of a circle bisect each other at O.

Then OA = OC and OB = OD

To prove:

(i) AC and BD are the diameters. In other words, O is the centre of the circle.

(ii) ABCD is a rectangle.

Proof: (i) In triangles AOD and BOC,

AO = OC [given]

$\angle$ AOD = $\angle$ BOC [Vertically opp.]

OD = OB [given]

$\therefore$ $\triangle$ AOD $\cong$ $\triangle$ COB [SAS congruency]

$\Rightarrow$ AD = CB [By CPCT]

Similarly, $\triangle$ AOB $\cong$ $\triangle$ COD

$\Rightarrow$ AB = CD

$\Rightarrow$ $\overset{\frown}{AB} \cong \overset{\frown}{CD}$ [Arcs opposite to equal chords]

$\Rightarrow$ $\overset{\frown}{AB} + \overset{\frown}{BC} \cong \overset{\frown}{CD} + \overset{\frown}{BC}$ $\Rightarrow$ $\overset{\frown}{ABC} \cong \overset{\frown}{BCD}$

$\Rightarrow$ AC = BD [Chords opposites to equal arcs]

$\therefore$ AC and BD are the diameters as only diameters can bisect each other as the chords of the circle.

(ii) Ac is the diameter. [Proved in (i)]

$\therefore \angle B = \angle D = 90^\circ$ ....(i) [Angle in semi-circle]

Similarly, BD is the diameter.

$\therefore \angle A = \angle C = 90^\circ$ ....(ii) [Angle in semi-circle]

Now diameters AC = BD

$\Rightarrow \overset{\frown}{AC} \cong \overset{\frown}{BD}$ [Arcs opposite to equal chords]

$\Rightarrow \overset{\frown}{AC} - \overset{\frown}{DC} \cong \overset{\frown}{BD} - \overset{\frown}{DC}$

$\Rightarrow \overset{\frown}{AD} \cong \overset{\frown}{BC}$

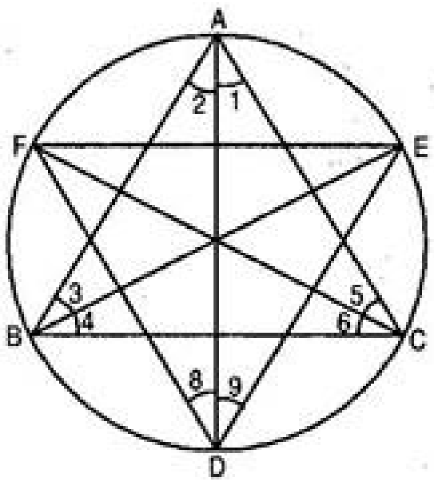
$\Rightarrow$ AD = BC [Chords corresponding to the equal arcs] ....(iii)

Similarly AB = DC ....(iv)

From eq. (i), (ii), (iii) and (iv), we observe that each angle of the quadrilateral is 90° and opposite sides are equal.

Hence ABCD is a rectangle.

6. Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F respectively. Prove that angles of the triangle are $\left(90^\circ - \frac{A}{2}\right)$, $\left(90^\circ - \frac{B}{2}\right)$ and $\left(90^\circ - \frac{C}{2}\right)$ respectively.



Ans. According to question, AD is bisector of $\angle A$.

$\therefore \angle 1 = \angle 2 = \frac{A}{2}$

And BE is the bisector of $\angle B$.

$\therefore \angle 3 = \angle 4 = \frac{B}{2}$

Also CF is the bisector of $\angle C$.

$\therefore \angle 5 = \angle 6 = \frac{C}{2}$

Since the angles in the same segment of a circle are equal.

$\therefore \angle 9 = \angle 3$ [angles subtended by $\overset{\frown}{AE}$] ....(i)

And $\angle 8 = \angle 5$ [angles subtended by $\overset{\frown}{FA}$] ....(ii)

Adding both equations,

$\angle 9 + \angle 8 = \angle 3 + \angle 5$

$\Rightarrow \angle D = \frac{B}{2} + \frac{C}{2}$

Similarly, $\angle E = \frac{A}{2} + \frac{C}{2}$ and $\angle F = \frac{A}{2} + \frac{B}{2}$

In triangle DEF,

$\angle D + \angle E + \angle F = 180°$

$\Rightarrow \angle D = 180° - (\angle E + \angle F)$

$\Rightarrow \angle D = 180° - \left(\frac{A}{2} + \frac{C}{2} + \frac{A}{2} + \frac{B}{2}\right)$

$\Rightarrow \angle D = 180° - \left(\frac{A}{2} + \frac{B}{2} + \frac{C}{2}\right) - \frac{A}{2}$

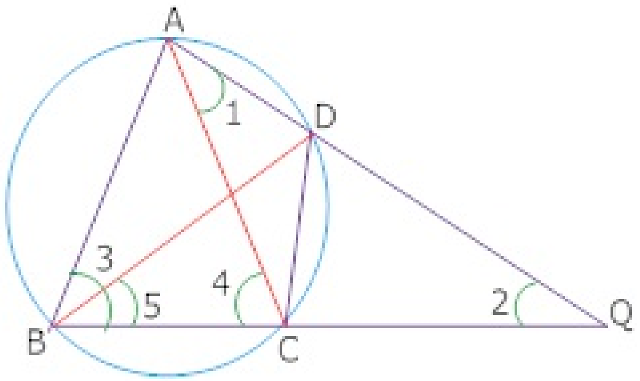
$\Rightarrow \angle D = 180° - 90° - \frac{A}{2}$ [$\because \angle A + \angle B + \angle C = 180°$]

$\Rightarrow \angle D = 90^{\circ} - \frac{A}{2}$

Similarly, we can prove that

$\angle E = 90^{\circ} - \frac{B}{2}$ and $\angle F = 90^{\circ} - \frac{C}{2}$

7. The bisector of $\angle B$ of an isosceles triangle ABC with AB = AC meets the circum circle of ΔABC at P if AP and BC produced meet at Q, prove that CQ = CA



Ans. Join P and C

Considered ΔACQ

$\angle 4 = \angle 1 + \angle 2 ....(i)$

[Exterior angle is equal to the sum of two interior opposite angles]

$\angle 4 = \angle 3 ....(ii)$

From (i) and (ii)

$\angle 1 + \angle 2 = \angle 3$

$\angle 3 = 2\angle 5$ [BP is bisector of $\angle 3$]

$\angle 1 + \angle 2 = 2\angle 5$

$\angle 1 + \angle 2 = \angle 5 + \angle 5$

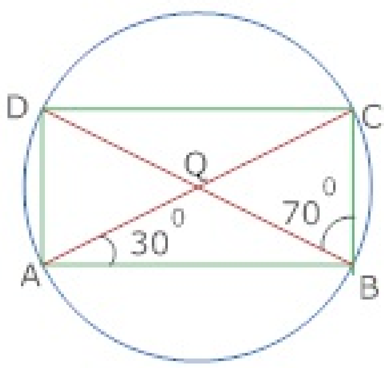
$\angle 5 = \angle 1$ [Angle in the same segments]

$\angle 1 + \angle 2 = \angle 1 + \angle 1$

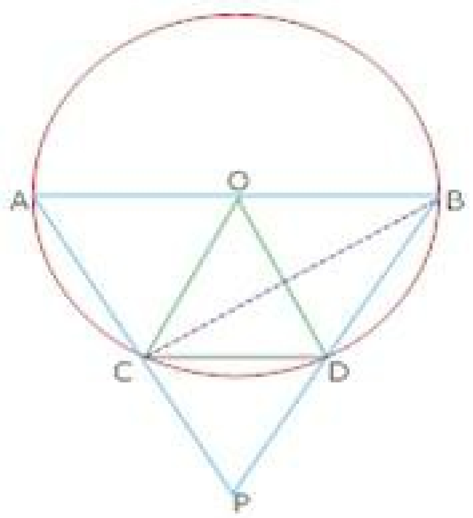
$\angle 2 = \angle 1$

In $\Delta ACQ,\ CQ = CA$ (sides opposite to equal angles)

8. OC radius equal to chord CD and AB is diameter and AC and BD produced meet at P so prove that $\angle CPD = 60°$



Ans. Join BC



In ΔOCD, $OC = OD$ (Radii of same circle)

OC=CD (Given)

OC=OD=CD

ΔOCD is equilateral

Hence, $\angle COD = 60^\circ$

$\angle CBD = 30^\circ$

[Angle subtended by arc CD at centre is double the angle at any Pont of the remaining part]

$\angle ACB = 90^\circ$

Exterior $\angle ACB = \angle CBP + \angle CPB$

$90^\circ = 30^\circ + \angle CPB$

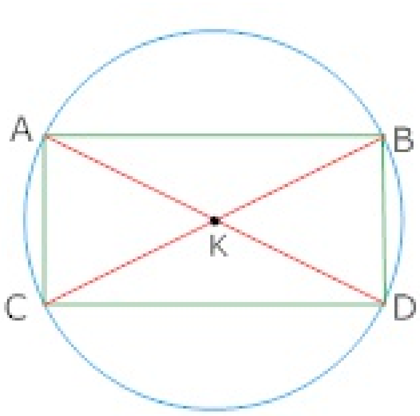
$\angle CPB = 60^\circ$

$\angle CPD = 60^\circ$

9. The two chords bisect each other AD and BC show that

(i) AD and BC are diameter

(ii) ABCD is a rectangle



Ans. Given that the two chords AD, BC of the circle bisect each other.

Let these cords bisect at K

In Δ AKB and Δ DKC

AK=DK [AB, CD bisect each other at K]

BK= CK

$\angle AKB = \angle DKC$ [Vertically opposite Δ]

$\Delta AKB \cong \Delta DKC$ (by SAS)

$AB = CD$ (CPCT)

$\overset{\frown}{AB} = \overset{\frown}{CD}$

$\overset{\frown}{AB} + \overset{\frown}{BC} = \overset{\frown}{CD} + \overset{\frown}{BC}$

$A\overset{\frown}{C} = B\overset{\frown}{D}$

$AC = BD$

Also, in quadrilateral ABCD

AB=CD

$\angle A = \angle B = \angle C = \angle D = 90^\circ$ [AC, BD is diameter so angle is semicircle)

10. Show that $\angle AHE$ and $\angle EGC$ are supplementary. Given that ABC AEG and HEC are straight lines.

Ans. $\angle H + \angle 1 = 180°$ [Opposite $\angle s$ of cyclic quadrilateral ABEH]..........(i)

$\angle G + \angle 2 = 180°$ [Opposite $\angle s$ of cyclic quadrilateral BCGE]..........(ii)

Adding (i) and (2) we get

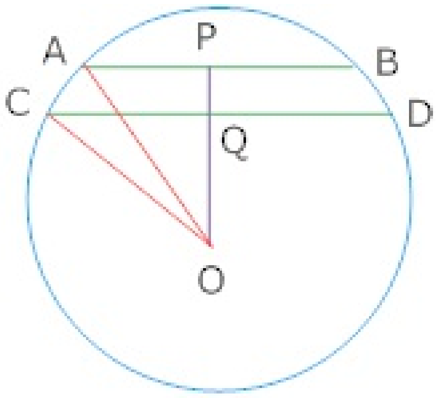
$\angle H + \angle G + \angle 1 + \angle 2 = 360°$

$\angle H + \angle G = 360° - (\angle 1 + \angle 2)$

$= 360° - 180°$ [$\angle 1 + \angle 2 = 180°$ Linear pair]

$= 180°$

11. OP $\perp$ AB, OQ $\perp$ CD, AB||CD. AB=6 cm and CD = 8 cm, Determine PQ, of circle of radius 5 cm.



Ans. Join OA and O

$AB = 6cm$

$AP = \frac{1}{2} AB$

AP=3 cm

CD=8 cm

$$CQ = \frac{1}{2} CD$$

CQ=4 cm

In right angled triangle ΔAPQ

$$AO^2 = PO^2 + AP^2$$

$$(5)^2 = PO^2 + (3)^2$$

$$PO^2 = 16$$

$$PO = \sqrt{16}$$

=4 cm

In rt. ΔOQC

$$CO^2 = CQ^2 + OQ^2$$

$$(5)^2 = (4)^2 + OQ^2$$

$$OQ = \sqrt{9}$$

= 3 cm

$$AB \parallel CD$$

$$\angle APO = \angle CQO$$

PO and QO are in the same line

PQ = PO – OQ = 4 – 3 = 1c